

## Simplified stability analysis of geosynthetic-reinforced embankment subjected to over-topping flow

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### ABSTRACT

Water-front or water-impounding embankments, such as river and coastal levees, tsunami barriers, fill dams etc. may collapse if the global stability of the downstream slope is lost by over-topping flow and/or seepage flow caused by flood and tsunami. To prevent the above, impervious slope protections, such as concrete panel facing or slab facing, are often employed. These slope protections become stable when connected to geosynthetic layers that reinforce the embankment body. Based on global safety factors evaluated by the modified Fellenius limit equilibrium circular slip stability analysis, the stability of an embankment downstream slope is analyzed. It is shown that the slope stability decreases with an increase in the depth of overtopping flow and/or the seepage force, whereas the stability increases effectively by the use of impervious slope protection connected to geosynthetic reinforcements.

**Keywords:** slope stability; over-topping flow; modified Fellenius slice method; seepage

### 1 INTRODUCTION

The downstream slopes of such embankments as river levees, coastal levees, tsunami barriers, fill dams, other types of reservoir etc. may collapse, often triggered by external or internal erosion and/or scouring, by deep over-topping flow and/or seepage caused by flood or tsunami, which may result in a total loss of cross-section (e.g., the Great East Japan Earthquake disaster, March 2011). To prevent the above, impervious slope protections, such as concrete panel facing and slab facing, are often arranged on the downstream slope. In addition, planar geosynthetic layers may be arranged which are reinforcing the embankment body and, being connected to the slope protection, stabilize the latter.

In this paper, effects of over-topping flow and seepage, as wells as the use of impervious slope protection and geosynthetic reinforcement, on the stability of embankment downstream slope are evaluated. To this end, equations for the global safety factors under various conditions were formulated based on the modified Fellenius limit equilibrium slice method. By performing a series of numerical analysis of a typical slope model, it is shown that the factor of safety decreases with an increase in the depth of over-topping flow and the seepage force, while it increases effectively by the use of impervious slope protection connected to geosynthetic layers reinforcing the embankment body. Analysis of external and internal erosion and scouring is beyond the scope of this study.

### 2 FORMULATION

To evaluate the individual or combined effects of the factors mentioned above, the six cases shown in Fig. 1 are analyzed. The equations relevant to these cases are formulated within the framework of the modified Fellenius method. In this method, it is assumed that, in each slice, the resultant of interslice effective earth pressures is always in parallel to the slice base (Fig. 2). The factor of safety  $F_s$  against circular slip (radius  $R$ ) in terms of overall moment equilibrium is defined as:

$$\begin{cases} F_s = [F_{\text{global}}]_{\min} = [M_r / M_d]_{\min} \\ M_r = R \cdot \sum [S_{fi}] = R \cdot \sum [c'_i \cdot l_i + P'_i \cdot \tan \phi'_i] \\ M_d = R \cdot \sum [S_{wi}] = R \cdot \sum (W_i \cdot \sin \alpha_i) - M_w \end{cases} \quad (1)$$

where  $M_r$  is the resisting moment;  $M_d$  the driving moment;  $S_{fi}$  the soil shear strength (cohesion  $c'_i$  & friction angle  $\phi'_i$ ) mobilized on the base of the  $i^{\text{th}}$  slice (angle  $\alpha_i$  & length  $l_i$ );  $S_{wi}$  the shear force acting on the slice base;  $P'_i$  the effective normal load on the slice base;  $W_i$  the total slice weight; and  $-M_w$  the moment due to the overburden water pressure  $U_{AB}$  acting on the submerged part of the downstream slope (Fig. 2).

#### 2.1 Case 1: Submerged slope without seepage and over-topping flow

In this case, Eq. 2 for  $F_s$  is obtained by substituting the following terms for  $P'_i$  and  $M_w$  into Eq. 1:  $P'_i = (W_i - W_{bi}) \cos \alpha_i = W'_i \cdot \cos \alpha_i$ ; and  $M_w = \sum W_{wi} \sin \alpha_i - U_{AD} \cdot y_h / R$ .

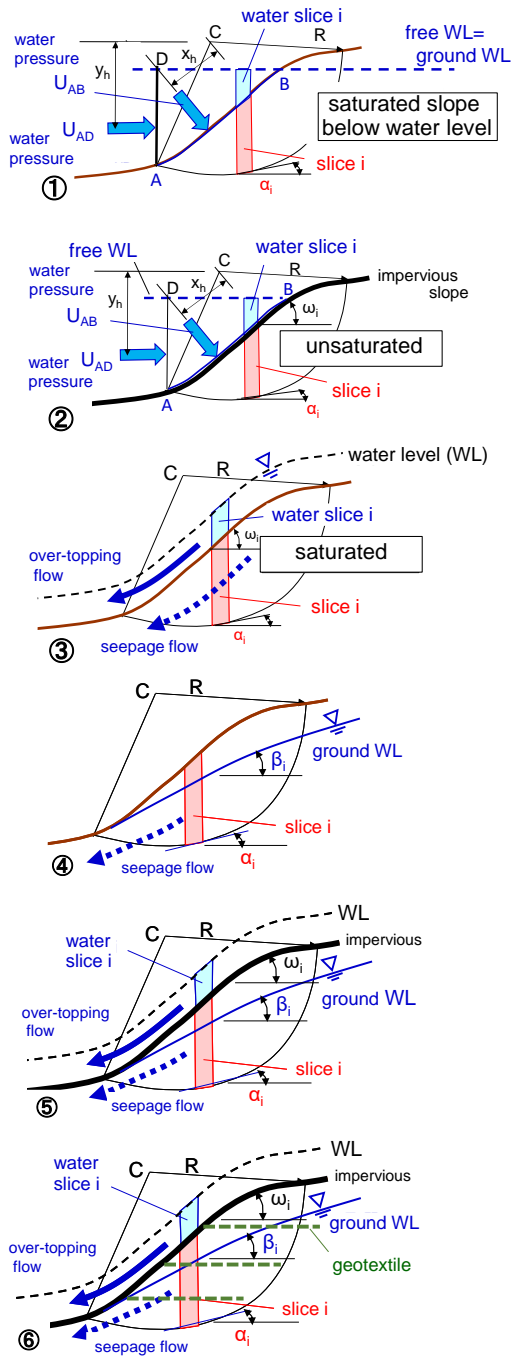


Fig. 1. Six cases of downstream slope analyzed in this study.

This normal load  $P'_i$  is obtained by assuming that the resultant of interslice effective earth pressures is in parallel to the slice base and by the fact that the interstitial water pressure is acting horizontal on the vertical slice boundaries (i.e. the modified Fellenius

method).  $W_{bi}$  is the buoyant force,  $W_{wi}$  the weight of the overburden water slice,  $U_{AD}$  the horizontal static water pressure resultant and  $y_h$  its vertical distance to the slip center (Fig. 1).

$$F_s = \frac{\sum [c'_i \cdot l_i + W'_i \cdot \cos \alpha_i \cdot \tan \phi'_i]}{\sum (W_i \cdot \sin \alpha_i) + \sum (W_{wi} \cdot \sin \alpha_i) - U_{AD} \cdot y_h / R} \quad (2)$$

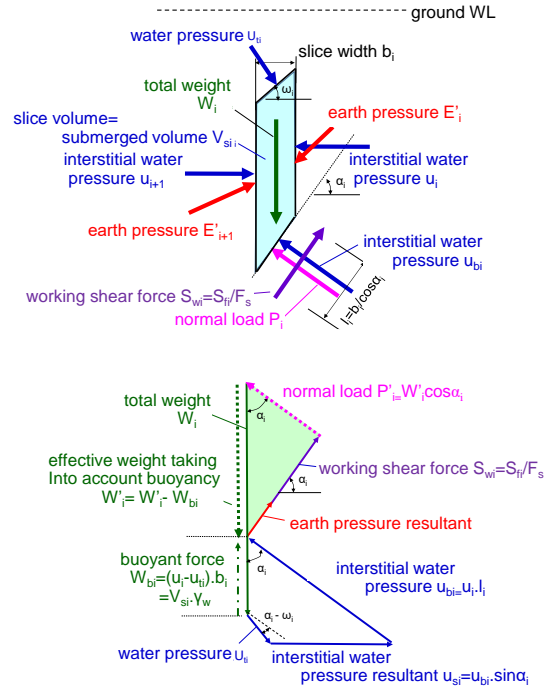


Fig. 2. Acting loads and forces polygon for a submerged slice without seepage and over-topping flow

### 2.2 Case 2: Slope with impervious slope protection and with static reservoir water

For an unsaturated slope covered by an impervious slope facing,  $F_s$  is given by Eq. 3, which is obtained by adding the beneficial effects of overburden water in the numerator of Eq. 2.

$$F_s = \frac{\sum [c'_i \cdot l_i + \{W_i \cdot \cos \alpha_i + \frac{W_{wi}}{\cos \omega_i} \cdot \cos(\alpha_i - \omega_i)\} \cdot \tan \phi'_i]}{\sum (W_i \cdot \sin \alpha_i) + \sum (W_{wi} \cdot \sin \alpha_i) - U_{AD} \cdot y_h / R} \quad (3)$$

where  $\omega_i$  is the slice top angle and  $W_{wi}/\cos \omega_i \cdot \cos(\alpha_i - \omega_i)$  is the component of the overburden water pressure  $W_{wi}/\cos \omega_i$  that is normal to the slice base. This component, combined with the term  $W_i \cdot \sin \alpha_i$  (where  $W_i$  is the total slice weight), greatly increases the slice normal load  $P'_i$ , thus the soil shear strength  $S_{fi}$ , thereby the  $F_s$  value, in comparison to case 1 (i.e., Eq. 2).

### 2.3 Case 3: Fully submerged slope subjected to over-topping flow & seepage

$F_s$  is given by Eq. 4, for which, in comparison to Eq. 2, terms accounting for over-topping flow and seepage are added to the numerator while the component for horizontal static water pressure  $U_{AD}$  is removed from

the denominator. Both of the above may largely decrease the  $F_s$  value.

$$F_s = \frac{\sum [c'_i \cdot l_i + [W_i \cdot \cos \alpha_i - W_{bi} \cdot \{\cos \alpha_i + \sin \omega_i \cdot \sin(\alpha_i - \omega_i)\}] \cdot \tan \phi'_i]}{\sum (W_i \cdot \sin \alpha_i) + \sum (W_{wi} \cdot \sin \alpha_i)} \quad (4)$$

$\omega_i$  is the seepage flow angle in slice  $i$ , which is the same as the angle  $\alpha_i$  at the top of each slice. Taking  $\alpha_i = \omega_i$ , the formula of the numerator of Eq. 4 returns to the one of the numerator of Eq. 2 (for the case with no over-topping flow and no seepage flow), whereas the over-topping flow height should be accounted for when evaluating the water slice weight  $W_{wi}$  in Eq. 4

#### 2.4 Case 4: Partially saturated slope subjected to seepage flow without over-topping flow

Eq. 5 is obtained by replacing  $\omega_i$  in Eq. 4 with the seepage flow angle  $\beta_i$  which is basically different from the slice top angle  $\omega_i$  and the slice base angle  $\alpha_i$ .

$$F_s = \frac{\sum [c'_i \cdot l_i + [W_i \cdot \cos \alpha_i - W_{bi} \cdot \{\cos \alpha_i + \sin \beta_i \cdot \sin(\alpha_i - \beta_i)\}] \cdot \tan \phi'_i]}{\sum (W_i \cdot \sin \alpha_i)} \quad (5)$$

The effect of seepage flow in the term including  $\sin(\alpha_i - \beta_i)$  may become either positive or negative depending on the sign of  $\alpha_i - \beta_i$  in respective slices.

#### 2.5 Case 5: Partially saturated slope with impervious slope protection subjected to over-topping flow & seepage

$F_s$  is given by Eq. 6, for which effects of overtopping flow are added to the numerator and denominator of Eq. 5 via the water slice weight  $W_{wi}$ .

$$F_s = \frac{\sum [c'_i \cdot l_i + [W_i \cdot \cos \alpha_i - W_{bi} \cdot \{\cos \alpha_i + \sin \beta_i \cdot \sin(\alpha_i - \beta_i)\}] + \theta_i \cdot W_{wi} \cdot \cos \alpha_i] \cdot \tan \phi'_i]}{\sum (W_i \cdot \sin \alpha_i) + \sum (W_{wi} \cdot \sin \alpha_i)} \quad (6)$$

$\theta_i$  represents the proportion of  $W_{wi}$  transmitted to the slice base increasing  $P^i$ . It is assumed that  $\theta_i$  is equal to the ratio of the volume of the unsaturated zone to the total slice volume,  $\theta_i = A_D / (A_D + A_W)$ .  $\theta_i = 1.0$  means that the slice is entirely unsaturated with  $W_{wi}$  fully increasing  $P^i$  at the slice base, similar to case 2; while  $\theta_i = 0$  means that the slice is entirely saturated with  $W_{wi}$  not increasing  $P^i$ , similar to cases 1 & 3.

#### 2.6 Case 6: Slope in case 5 that is geosynthetic-reinforced

Effects of tensile forces in the reinforcement layers are simply added to the numerator of Eq. 6 (case 5).

$$\left\{ \begin{aligned} F_s &= \frac{\sum [c'_i \cdot l_i + [W_i \cdot \cos \alpha_i - W_{bi} \cdot \{\cos \alpha_i + \sin \beta_i \cdot \sin(\alpha_i - \beta_i)\}] + \theta_i \cdot W_{wi} \cdot \cos \alpha_i] \cdot \tan \phi'_i + \sum T_{ri}}{\sum (W_i \cdot \sin \alpha_i) + \sum (W_{wi} \cdot \sin \alpha_i)} \\ T_{ri} &= T_i \cdot \sin \alpha_i + T_i \cdot \cos \alpha_i \cdot \tan \phi'_i \quad ; \quad T_i = \min \{ T_{ki} ; T_{pi} = 2 \int \sigma'_{vi} \cdot \tan \phi'_i \cdot dx \} \end{aligned} \right. \quad (7)$$

$T_{ri}$  is the resistance mobilized along the slice base (angle  $\alpha_i$ ) by the  $i^{\text{th}}$  reinforcement layer having the allowable strength  $T_i$ .  $T_i$  is the minimum of the

allowable tensile rupture strength  $T_{k,i}$  and the allowable pull out strength  $T_{p,i}$ . For  $T_{p,i}$ , the soil/reinforcement interface friction angle is assumed to be equal to the soil internal friction angle  $\Phi'$ ;  $\sigma'_{vi}$  is the effective overburden pressure; and  $x$  is the reinforcement longitudinal axis ( $= 0$  at the slip surface).

Further details on the derivation of Eqs. 2 to 7 are given in Tatsuoka & Duttine (2007a,b).

### 3 ANALYSIS OF MODEL SLOPE

Eqs. 2 to 7 were implemented in the stability analysis computer code developed by Duttine et al. (2018). The stability of a typical embankment slope (Fig. 3) was analyzed, in which the height is 15 m; the slope 1:2; and the soil unit weight & strength  $\gamma_i = 19 \text{ kN/m}^3$ ,  $\gamma_{sat} = 20 \text{ kN/m}^3$ ,  $c' = 6 \text{ kPa}$  &  $\Phi' = 40^\circ$ . These values are typically used in the slope stability analysis following the Japanese railways design codes. In case 6, a typical reinforcement rupture tensile strength  $T_k = 30 \text{ kN/m}$  was used. The hydraulic conditions in cases 1 - 6 are depicted in Fig. 3.

Two sets of analysis were performed in cases 1 - 6 (Table 1). In the first set (A), to confirm the stability of the analysis, the safety factor was obtained for a fixed circular slip that is the critical circular slip, exhibiting the minimum  $F_s$ , in case 1 (Fig. 3). In the second set of analysis (B), the critical circular slip exhibiting the minimum  $F_s$  was obtained in each case (Fig. 4).

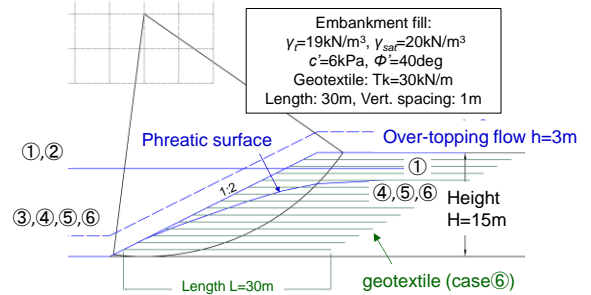


Fig. 3. Critical circular slip showing the minimum  $F_s$  in case 1, used in all the other cases (analysis set A):

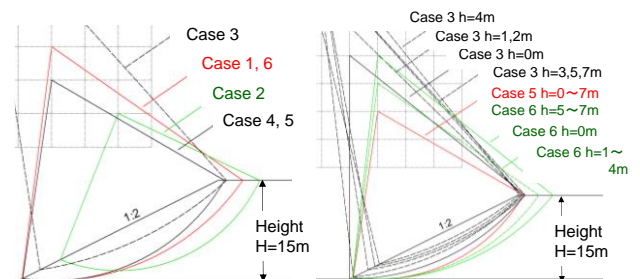


Fig. 4. (left) Critical circular slips showing the minimum  $F_s$  in each of cases 1 to 6 ( $h = 3 \text{ m}$ ); and (right) those for different over-topping depths  $h$  in cases 3, 5 & 6 (analysis set B):

Fig. 5 compares the  $F_s$  values obtained in cases 1 - 6 by analysis set A (Fig. 3) and set B (Fig. 4). It may be seen that the  $F_s$  value by set B is always smaller than

the one by set A. By comparing the  $F_s$  values in cases 2 to 6 with the one in case 1 (a slope submerged with static water) and with each other, the effects of the influencing factors can be evaluated. Case 2: the impervious slope protection that makes the slope entirely unsaturated drastically increases the  $F_s$  value. Case 4: Internal seepage significantly decreases the  $F_s$  value. Case 3: the  $F_s$  value decreases by the overtopping flow, to a value less than unity in this case where a saturated slope does not have impervious slope protection. Case 5: compared with case 3, the  $F_s$  value noticeably increases by the presence of impervious slope protection that creates an unsaturated zone immediately below the slope surface protection. Case 6: compared with case 5, the  $F_s$  value further increases by the use of geosynthetic reinforcement.

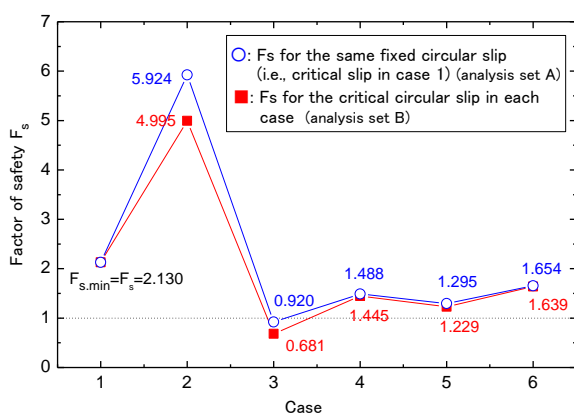


Fig. 5.  $F_s$  values for each case (for the model shown in Fig. 3)

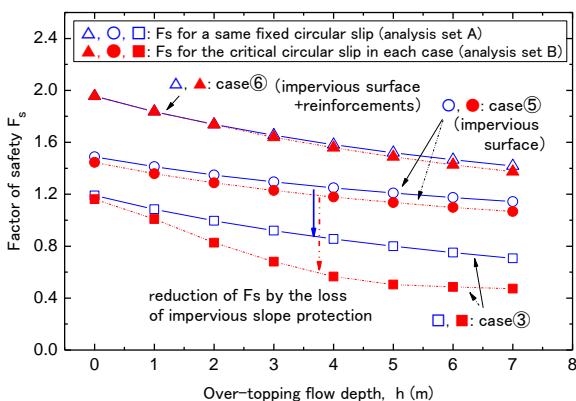


Fig. 6. Relation between  $F_s$  values and over-topping flow height

Fig. 6 shows the relationships between the  $F_s$  values obtained by analysis sets A and B and the overtopping flow height  $h$  in the cases 3, 5 and 6. In any of these cases, the  $F_s$  value consistently decreases with an increase in  $h$ . With the use of impervious slope protection (case 5), the  $F_s$  value is maintained larger than unity even when  $h$  reaches 7m. With the additional use of reinforcement (case 6), the  $F_s$  value when  $h=7m$  is as high as around 1.4. In contrast, in case 3 (without impervious slope protection), the  $F_s$  value drops quickly with an increase in  $h$ , with  $F_s$  (set B) becoming

nearly unity already when  $h=1m$ . In case 5, the impervious slope protection may be easily lost, as it is not connected to reinforcement layers as in case 6. Thus, the  $F_s$  value may quickly drop to the value in case 3. Besides, in case 3, it is much more likely than in case 6 that external and/or internal erosion, and/or scouring, take place, resulting into a further decrease in the  $F_s$  value from those (set B) presented in Fig. 6.

When the computed value of  $F_s$  becomes less than unity, the static equilibrium is lost and the slip starts exhibiting residual shear displacements. In cases 3, 5 and 6, when the time history of over-topping flow depth  $h(t)$  is provided, the residual slip displacements can be computed based on Eqs. 4, 6 and 7, similarly as in a Newmark sliding block seismic analysis.

#### 4 CONCLUSIONS

To evaluate the effects of over-topping flow, internal seepage, as well as the use of impervious slope protection and geosynthetic reinforcements on the stability of an embankment downstream slope, a series of stability analysis was performed based on the safety factor equations formulated in the simple framework of the modified Fellenius slice stability analysis. A typical 15m-high embankment slope was analyzed. It was confirmed that the factor of safety decreases by the over-topping flow and internal seepage, whereas it increases by the use of impervious slope protection and further by the use of geosynthetic reinforcement layers connected to the slope protection. These results indicate that the combined use of impervious slope protection and geosynthetic reinforcement is very effective to protect the embankment slope against the over-topping flow and seepage flow.

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